Formulas for the Final Exam

- Return of a security following the single index model

$$r_i = E(r_i) + \beta_i F + e_i$$

- Variance of a security following the single index model

$$Var(r_i) = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

- Covariance between two risky securities each following the single index model

$$Cov(r_i, r_j) = \beta_i \beta_j \sigma_M^2$$

- Single factor APT equation

$$\frac{E(r_A) - r_f}{\beta_A} = \frac{E(r_B) - r_f}{\beta_B}$$

- Multiple factor APT equation

$$E(r_P) = r_f + \beta_{P1}[E(r_1) - r_f] + \beta_{P2}[E(r_2) - r_f] + ... + \beta_{Pn}[E(r_n) - r_f],$$

  where r_i, $i = 1, 2, ..., n$ is a return of a factor portfolio i

- Realized compound yield

$$RCY = \left(\frac{FV}{P}\right)^{1/n} - 1$$

- Forward rate between years $n-1$ and n:

$$(1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n)$$

- Yield of an n–year zero–coupon bond in terms of annual forward rates

$$(1 + y_n)^n = (1 + y_1)(1 + f_2)(1 + f_3)...(1 + f_n)$$

- Forward rate between years $n-k$ and n in terms of annual forward rates:

$$f_{n-k,n} = (1 + f_{n-k+1})(1 + f_{n-k+2})...(1 + f_n) - 1$$

- Forward rate according to the liquidity preference hypothesis

$$f_n = r_n + \Delta_n$$

- Bond duration:

$$D = \sum_{i=1}^{n} t_i \times w_i, \quad \text{where} \quad w_i = \frac{CF_i/(1+y)^i}{P},$$

- Bond Convexity:
$$\text{Convexity} = \frac{(\Delta t)^2}{P \times (1+y)^2} \sum_{i=1}^{n} \left[\frac{CF_i}{(1+y)^i} (i^2 + i) \right]$$
where Δt is a time between two consequent payments of a bond
- Percentage price change of a bond:
$$\frac{\Delta P}{P} = -D \times \frac{\Delta(y^a)}{1+y} + \frac{1}{2}\text{Convexity} \times [(\Delta y^a)]^2$$
- Duration of a level annuity
$$\Delta t \left(\frac{1+y}{y} - \frac{n}{(1+y)^n - 1} \right)$$
- Duration of a corporate bond
$$\Delta t \left(\frac{1+y}{y} - \frac{(1+y) + n(c-y)}{c[(1+y)^n - 1] + y} \right),$$
- Duration of a level perpetuity is
$$\Delta t \, \frac{1+y}{y}$$
- Put–Call Parity
$$P_0 + S_0 = C + \frac{X}{1+r_f} + PV(D)$$
- Price of European vanilla option in one–period binomial model:
$$C = \frac{C_u(1-d+r_f) + C_d(u-1-r_f)}{(u-d)(1+r_f)},$$
Or
$$C = (C_u P_u + C_d P_d)/(1+r_f),$$
where $P_u = (1-d+r_f)/[(u-d)]$ and $P_d = (u-1-r_f)/[(u-d)]$
- Hedge ratio and dollar position in bonds in the replicating portfolio:
$$H = \frac{C_u - C_d}{S_0(u-d)}$$
$$ZB_0 = \frac{C_d u - C_u d}{(u-d)(1+r_f)}$$
- Price of American call option at node n of the binomial tree
$$C_n^A = max\{(S_n - X)^+, C_n\},$$
where S_n is the price of a stock at this node and C_n is the price of an option if it is not exercised given by
$$C_n = \frac{C_u^A(1-d+r_f) + C_d^A(u-1-r_f)}{(u-d)(1+r_f)},$$
where C_u^A, C_d^A are the prices of the American option in the following upper and lower nodes.